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THE APPLICABILITY OF SAINT-VENANT'S PRINCIPLE TO MONOCOQUE STRUCTURES

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An extremely strong effect of boundary conditions on the character of damping of the stressed-strained state in a cylindrical envelope with a free edge under the action of concentrated normal forces transferred through an elastic frame has been revealed. Thus, under the effect of radial load at a large distance from the free edge of the envelope, the values of normal displacement and annular bending moment not only decrease on approaching the free edge but also increase, reaching a maximum value on the free edge of the envelope, which is in conflict with Saint-Venant's principle.

1. Modern flying vehicles are usually designed according to the principle of a reinforced monocoque. The fuselage of an airplane is a circular or almost circular thin-walled cylinder strengthened by uniformly arranged rings (frames) and longitudinal rigidity elements (stringers). This, to a large extent, is characteristic of other types of flying vehicles.

All rigidity elements are positioned on the inner surface of the cylinder such that the outer surface remains smooth. As a rule, these elements are continuous and longitudinal elements pass through the holes cut in annular elements. For many years, stresses in the fuselage of the reinforced monocoque type were calculated by the commonly adopted beam flexural theory, which is based, as is known, on the hypotheses of Euler, Bernoulli, and Navier, according to which the cylinder cross sections that are perpendicular to its axis before bending remain the same as after load application and the shape of the cylinder cross section is not distorted in its plane during loading. In most problems related to structures with a rigid cross section, for shear stresses and bending the formulas obtained from the elementary beam theory are rather accurate. Deviation from the results of this theory is most noticeable near the places where concentrated or localized loads are applied and, by its nature, this deviation has a local character, existing mainly in the region which in size is comparable with the cross section. The fact that the phenomenon has a local character was found by B. Saint-Venant [1] and was confirmed by J. N. Goodier [2]. Later, N. J. Hoff [3] and V. L. Salerno [4] found that the size of the region where the effect of the concentrated force on the reinforced structure of the monocoque type manifests itself is comparable with the cylinder diameter.

The phenomena related to deviation from Saint-Venant's principle not only in thin-walled rods of open contour but also in thin-walled envelopes were mentioned by V. Z. Vlasov in his works where experimental results were discussed [5]. This prompted the authors of [6] to conduct theoretical and experimental studies of the applicability of Saint-Venant's principle to envelopes of zero curvature. On the basis of the semi-zero-moment theory they first obtained results under the effect of force and temperature fields with piecewise-continuous distribution along the generatrix and cosinusoidal distribution around the circumference. In this work, we continue consideration of the problem of the circular cylindrical envelopes affected by concentrated normal forces transferred through an elastic frame that is rigid to bending in its plane but is elastic from the plane.

2. The stressed-strained state of the envelope is determined by the modified equations of the semi-zero-moment theory, which, as is known, describe well the so-called basic state of the envelope under the effect of loads with limited variability along the contour, e.g., loads of the type $p(\alpha, \beta) = p(\alpha) \cos n\beta$, where $n < n^*$ [7].

The resolving equation for the basic stressed state of the circular cylindrical envelope has the form

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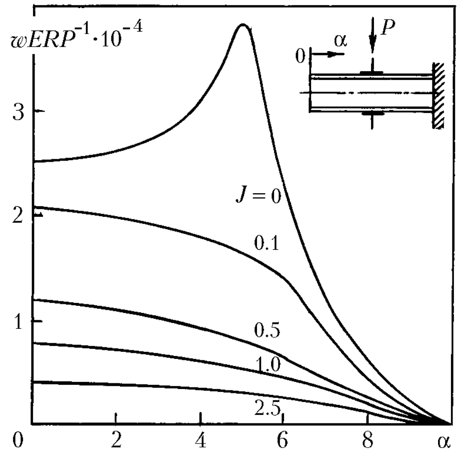

Fig. 1. Variation of normal displacement along the zero generatrix of the envelope (β = 0) under the action of radial concentrated load P transferred at the center of the envelope ($\alpha = x/R = 5$) through the elastic frame ($J \neq 0$) or directly to the envelope (J = 0).

$$\frac{\partial^4 \Phi}{\partial \alpha^4} + \frac{c^2}{1-\nu^2}\frac{\partial^4}{\partial \beta^4}\left(\frac{\partial^2}{\partial \beta^2}+1\right)^2 \Phi = 0\,, \tag{1}$$

where $\Phi(\alpha, \beta)$ is the resolving function, $c^2 = h^2/12R^2$.

Displacement, forces, and bending moments are related to the resolving function by differential equations:

$$u = -\frac{\partial^3 \Phi}{\partial \alpha \partial \beta^2}\,, \quad v = \frac{\partial^3 \Phi}{\partial \beta^3}\,, \quad w = \frac{\partial^4 \Phi}{\partial \beta^4}\,;$$
$$T_1 = -\frac{Eh}{R}\frac{\partial^4 \Phi}{\partial \alpha^2 \partial \beta^2}\,; \quad S = \frac{Eh}{R}\frac{\partial^4 \Phi}{\partial \alpha^3 \partial \beta}\,; \quad G_2 = -D\left(\frac{\partial^6 \Phi}{\partial \beta^6}+\frac{\partial^4 \Phi}{\partial \beta^4}\right); \quad G_1 = \nu G_2\,. \tag{2}$$

Here, D is the cylindrical rigidity of the envelope.

The problem of frame bending in its plane can be reduced to solution of the differential equation relative to radial displacement $w(\beta)$:

$$\frac{d^5 w}{d\beta^5} + 2\frac{d^3 w}{d\beta^3} + \frac{dw}{d\beta} = \frac{R^4}{EJ}\left(\frac{dP}{Rd\beta} - q\right), \tag{3}$$

where EJ is the flexural rigidity of the frame in its plane under the action of radial load P.

Expressing the resolving function $\Phi(\alpha, \beta)$, displacements, forces, and bending moments in terms of the series over the circumferential coordinate β and formulating the conditions of equality, along the line $\alpha = \xi$, of the radial displacement of the frame and the envelope $w_e(\xi, \beta) = w_f$, we obtain the solution of the problem posed [8]. For the shell this solution is written by the method of initial parameters developed by V. Z. Vlasov as applied to calculation of envelopes under the effect of arbitrary loads. This method is most effective in the case of concentrated and local loads. We note that the problem of the applicability of this solution in the absence of the frame is not discussed in either the work mentioned or in the present work. However, if we restrict ourselves to calculation of displacements only, this solution is of profound interest, since it allows rather accurate description of the strained state of envelopes under different boundary conditions.

The effect of radial load on the stressed-strained state was analyzed for envelopes of different lengths and relative thickness under various boundary conditions. Moreover, the frame rigidity varied within a wide range. An ex-

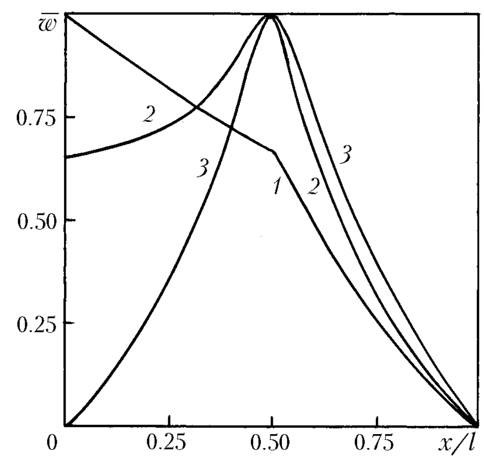

Fig. 2. Influence of the boundary conditions of the envelopes and the rigidity of the frames on the character of the behavior on normal displacement $\overline{w}$ along the zero generatrix (β = 0) under the action of a radial concentrated force P through the frame at the center of the envelope: 1 and 2) envelope with a free edge and a rigidly pinched edge; 3) envelope with both edges hinge fixed.

tremely strong influence of the boundary conditions on the character of damping of the stressed-strained state is revealed. At the same time, for example, we know about the complete absence of the effect of boundary conditions on the length of the zone of damping of a simple boundary effect. Then, under the action of radial load at a large distance from the free edge of the envelope, values of normal displacement and annular bending moment near the free edge not only decrease but even increase, reaching a maximum value on the free edge of the envelope. This is rather clearly demonstrated by the curves in Fig. 1 for the following dimensions of the envelope and frames: R = 0.2 m; R/h = 100; $J = \overline{J}\cdot 10^{-8}$ m^4, where $\overline{J}$ is equal to 0, 0.1, 0.5, 1.0, and 2.5.

3. By virtue of the fact that the analysis of structures by the finite-element method is at present the world's standard for strength and other types of calculation of structures, it is of interest to make calculations based on the finite-element method.

The structure is the finite-element model of a circular cylindrical envelope of finite length that at the center is strengthened by an elastic frame via which a radial concentrated force is transferred. The envelope model consists of 544 elements of the Plate type and 533 nodes, the material is steel with the Young module $E = 2\cdot 10^4$ daN/mm^2, the thickness is h = 2 mm, the length is 4000 mm, and the radius is 2000 mm. The frame model has 32 elements of the Beam type, and the material is steel. Two versions of the size of the rectangular cross section of the frame were considered: height along the radius — 40 and 8 mm, width — 2 mm. Three types of boundary conditions of envelopes were realized: a free edge, a hinge-fixed edge, and a rigidly pinched edge. In this case, one envelope has a free edge (x = 0) and a rigidly pinched edge (x = 1); in the other envelope, both edges are hinge fixed. Results of the calculation of the radial displacement for them are given in Fig. 2 in the form of curves 1 and 2 (free and rigidly pinched edges) and curve 3 (two hinge-fixed edges). The ratio of the normal displacement in the current cross section x/R to the maximum value of displacement $\overline{w} = w(x/R)/\max w$ is plotted on the ordinate axis. Curve 1 refers to the case where the concentration load is transferred to the envelope via the frame with a height of 40 mm and curve 2 — where the load acts through the frame having a height of 8 mm. Correspondingly, in the first case, the displacement has a maximum value on the free edge of the envelope and in the second case, at the center in the place of force action. In the same figure, curve 3 gives similar information for the same envelope but with both edges being hinge fixed. The concentrated load is transferred to this envelope at the center via the frame, as is the case with the first envelope. The principal difference in the behavior of curves 1 and 2 and curve 3 is quite obvious.

Thus, information obtained by the finite-element model confirms the conclusions drawn on the basis of the analytical solution.

NOTATION

E, elasticity modules of the envelope material, daN/mm^2; h, envelope thickness, mm; J, moment of inertia of the frame cross section, mm^4; l, envelope length, mm; n, number of the harmonic; P, radial concentrated force transferred to the envelope through the frame, N; p, distributed radial load to the envelope daN/mm^2; R, envelope radius, mm; x, longitudinal coordinate in the envelope, mm; α, dimensionless longitudinal coordinate; β, dimensionless circumferential coordinate; ν, Poisson coefficient of the envelope material. Indices: e, envelope; f, frame.

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